Planning Distribution Primary Feeders for Smart-Grid Operation via Network Flow Analysis

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***Abstract*—Planning the expansion of distribution systems (DS) is a nonlinear and combinatorial problem that combines technical and regulatory constraints. Commonly, the planning of DS is intended to achieve a radial topology to reduce its complexity. However, in a smart-grid context, distribution feeders are subject to reconfigurations to transfer load between feeders in failure events, or to compensate for voltage profile and power quality issues caused by distributed generation (DG). In this paper we propose a novel methodology for the planning of primary feeders, which considers the DS performance in both open and closed-loop arrangements. To this end we introduce the concept of reach current, which we use to define and solve a minimum power-loss flow problem. We solve this problem for each type of conductor considered to find a set of primary-feeder candidates. Additionally, an efficiency evaluation is performed to select the best among the candidate primary feeders. Simulation results on a test system show how this method is able to capture open and closed-loop operations, explicitly considering DG in the DS expansion planning within a smart-grid scheme.**

***Index Terms*—Distributed generation, Distribution systems planning, Network flow, Smart-grids**

I. Introduction

One of the challenges when planning the expansion of distribution systems (DS) is to find the network topology that best ensures that distribution utilities can offer a good quality of service while minimizing operating costs [1]. The expansion of DS is achieved by combining available corridors for the construction of lines, obtaining radial and mesh network topologies. Mesh networks are configured in open-loop or closed-loop arrangements, depending on the expected load and the required automation level, as different areas may have different power quality and reliability. In these arrangements, two primary feeders are linked to give support to each other during failure events, while the radial operation is ensured by the use of a normally-open switch. The primary feeder is the main structure of a distribution circuit, through which all demand nodes are connected to the substation.

Network reconfiguration is an efficient mechanism for service restoration and maintenance. In case of a failure in the primary feeder, the network is rearranged by changing the normally open and closed state of switches, while a portion of the load carried by the failed feeder is transferred to healthy feeders with remaining load capacity, reducing the energy non-supplied due to interruptions. Therefore, the foreseen open and closed-loop arrangements should be considered in DS expansion planning, in order to ensure the good performance of the DS under normal and contingency operational conditions.

In recent years, extensive research on DS planning has provided different methodologies to tackle the planning problem, including knowledge-based techniques, heuristics, and mathematical programming methods [1], [2]. Most of the previous work can be divided into radial system expansion [3] and meshed system reconfiguration [4], [5]. However, the interaction between open and closed-loop arrangements has not been considered as part of the expansion planning problem, as these topology rearrangements are normally treated as an operational problem. However, combining radial and meshed operations into a single methodology for expansion planning is essential to move forward into smart-grids schemes, where the network rearranges itself to respond to failure events.

In this paper, we propose a minimum power-loss flow approach to design primary feeders, considering open and closed-loop arrangements. For this purpose, we introduce the concept of *reach current* to model the power flow in lines. This approach allows us to consider technical and regulatory constraints, obtaining the set of lines that minimize the energy losses. The topology found can be used as a basis for radial expansion of the DS, while the mesh design ensures the necessary conditions for network reconfigurations.

We complement this method with an efficiency-based analysis, based on data envelopment analysis (DEA) [6], as a support tool to evaluate the primary feeders under multiple scenarios of operation and demand. In fact, the DS operating conditions are highly variable under a smart-grid scheme, which includes distributed generation (DG) and automation equipment. The power injection of DG or the behavior of demand under demand-response programs, could cause congestion or discharges in line sections. We make use of the efficiency analysis to evaluate a large set of DG and demand scenarios, allowing us to select those that offer the best results in terms of both technical and economic metrics.

The results show that this method is able to find the best primary feeders to operate under open and closed-loop arrangements. Together with the DEA-based evaluation methodology, the proposed method optimizes the final topology without

This work has been supported in part by Proyecto SILICE III, Codensa S.A. ESP - Department of Science, Technology and Innovation (Colciencias), Colombia, 2012-2014.

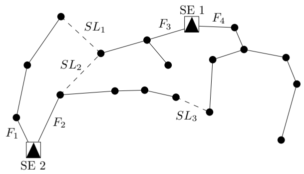


Fig. 1. Distribution primary feeders planning.

limiting it to be radial or meshed. Additionally, it identifies the most suitable configuration of the primary feeders according to the variable conditions of demand and DG penetration.

II. Primary Feeders Planning

We start this section with an example that illustrates the additional challenges faced when considering open and closed-loop arrangements as part of the DS expansion problem, as the operational conditions caused by the planning decisions need to be taken into account. Figure 1 shows how the connection of support feeders, from two different sources, depends on the selection of the connection point. For instance, in case of failure, feeder F_3 can be supported by feeder F_1 through open line SL_1, or by feeder F_2 through open line SL_2. The open lines can work in both directions in the sense that failure events can occur in both primary and support feeders. Moreover, the effective load that can be served by a support feeder depends on the demand of the load nodes, the capacity of the substation, and conductor type chosen. We therefore propose a DS planning methodology that considers these factors to find the best network topologies.

As summarized in Algorithm 1, the first step consists of finding a set of candidate primary feeders, between two substations, which is done by solving a minimum power-loss problem for each conductor type and pair of substations. Here we introduce the concept of *reach current*, which is the current that can be carried on the line without violating the voltage-drop limits. The reach current allows us to consider the effect of the conductor's characteristics on the ability of the lines to transport power and the associated power losses, and to consider DG as long as it remains below the demand, keeping the DS substations as a net source. Further, for each of these primary-feeder candidates, we can estimate its associated capital costs (x_1), and its operating scenarios, which arise by considering all the possible open and closed-loop arrangements along the primary feeder, i.e., changing the state of normally open and closed switches in the network. The next step is to determine the power losses and the feeders' chargeability for each of these scenarios, which is done by means of a power flow analysis. We summarize the results of these scenarios by using the 99-th percentile of the power losses observed (x_2), and the average chargeability (x_3). We make use of the average chargeability to evaluate whether the utilization of the lines' capacity stays within the expected limits. Finally, we use the DEA-based analysis proposed in [7] to find the best feeders among the set of candidates F. Based on the results, we determine the most suitable open-loop arrangement for the feeder, using the relative frequency of the scenario with fewer power losses.

Algorithm 1: Pseudocode of Primary Feeders Planning

Data: DS features and decision variables
Input: Graph R, conductor types s_t, substations, $F = \emptyset$
for *Each pair of substations* **do**
  for $i \leftarrow 1$ **to** c_t **do**
    $F \leftarrow F \cup$ Solution minimum power-loss flow problem
for $f_i \in F$ **do**
  $x_1^i \leftarrow$ Capital costs
  ${n_s}^i \leftarrow$ *Operating scenarios of feeder* i
  for $j \leftarrow 1$ **to** ${n_s}^i$ **do**
    ${x_2}_j^i \leftarrow$ Power losses in scenario j
    ${x_3}_j^i \leftarrow$ Chargeability in scenario j
  ${x_2}^i = P_{99}[{x_2}_j^i] \leftarrow$ Power losses
  ${x_3}^i = \frac{1}{{n_s}^i} \sum_{j=1}^{{n_s}^i} {x_3}_j^i \leftarrow$ Chargeability
Efficiency analysis: $\{(x_1^i, x_2^i, x_3^i), f_i \in F\}$
Relative frequency $F_{i,j}$ - scenario j of fewer power losses

A. Minimum Power-Loss Flow Problem

To find the main feeder candidates between each pair of substations, we represent the DS as a graph $R = (V, E, p^{loss}, u)$, where V denotes the set of nodes with cardinality $n = |V|$ that includes load, transfer, and substation nodes. The set of candidate lines is denoted by E. We define the function of power losses p_{ij}^{loss}, and the function of power transmission capacity u_{ij}, associated to each line (i, j). The minimum power-loss flow problem is formulated as an optimization program [8],

$$\begin{aligned} \min \; & \sum\nolimits_{(i,j)\in E} p_{ij}^{loss}(a_{ij}) \\ \text{s.t.} \; & \sum\nolimits_{j:(i,j)\in E} a_{ij} - \sum\nolimits_{j:(i,j)\in E} a_{ji} = b_i, \; \forall i \in V, \\ & a_{ij} \leq u_{ij}, \; \forall (i,j) \in E \end{aligned}$$

where a_{ij} is the variable of power, in watts [W], flowing through lines (i, j). Each node i has an associated net power flow, reflecting its power demand or supply. We define one substation as the source, and assign it a supply equivalent to the total demand in the DS, while the other substation is the sink, with a matching demand. All the other nodes are defined as transfer nodes with $b_i = 0$. This allows us, by solving the minimum power-loss problem, to find a single path between the substations with the capacity to support the peak demand considered. The functions p_{ij}^{loss} and u_{ij} depend on the conductor selected and its reach current, defined below.

The reach of a conductor [1] is defined, as the distance up to which the power on the line can be transported without violating the voltage-drop limits. Based on this definition, we use the DS steady-state representation to introduce the concept of *reach current*. For 3-phase circuits, with resistance R_{ij} and reactance X_{ij}, in [Ohms/km], and a current I_{ij} thats flow on line (i, j), the voltage drop line-to-line is

$$\Delta V_{ij} = \sqrt{3} I_{ij} (R_{ij} \cos\theta_{ij} + X_{ij} \sin\theta_{ij}) l_{ij},$$

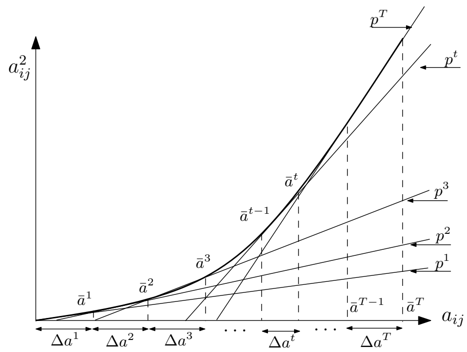


Fig. 2. Piecewise linear function of a_{ij}.

where l_{ij} is the length of the line (i, j), and $\cos\theta_{ij}$ is the power factor. Therefore, if the permissible voltage drop is ΔV_{ij}, the reach current of a conductor in line (i, j) is defined as

$$I_{ij}^r = \frac{\Delta V_{ij}}{\sqrt{3}(R_{ij}\cos\theta_{ij} + X_{ij}\sin\theta_{ij})l_{ij}}.$$

Based on this expression, we determine the power transmission capacity u_{ij} of a conductor on line (i, j), defined as

$$u_{ij} = \sqrt{3} I_{ij}^r V_i \cos\theta_{ij}.$$

The limit u_{ij} represents the maximum feasible power flow on the lines. Since the problem is solved for the *net* power flow b_i, any level of DG penetration can be considered, as long as it remains below the peak DS load.

Also, the power-loss function p_{ij}^{loss}, which represents the losses due to the energy dissipated in the conductors, expressed in terms of the power a_{ij}, can be expressed as

$$p_{ij}^{loss}(a_{ij}) = 3I_{ij}^2 R_{ij} l_{ij} = \frac{R_{ij} l_{ij}}{(V_i \cos\theta_{ij})^2} a_{ij}^2,$$

using the real power equation in a balanced three-phase system. As a result, the minimum power-loss flow problem has a quadratic objective function.

As the optimization problem is quadratic, we use a piecewise linear model to approximate the quadratic power flow by a linear function [9]. Piecewise linearization methods have been widely applied to problems in engineering, including the minimum cost network flow problem. A good piecewise linearization method must consider the effective break points selection strategy to enhance the computational efficiency [10]. As shown in Figure 2, the curve a_{ij}^2 is approximated by a series of straight lines, dividing the curve in T intervals of length Δa^t. Each interval has upper and lower bounds, denoted by $\bar{a}^t$ and $\bar{a}^{t-1}$, respectively. The value of the power in each interval, denoted by a^t, is limited to be within each interval as $\Delta a^t = \bar{a}^t - \bar{a}^{t-1}$. The power flow a_{ij}^2 takes the form

$$a_{ij}^2 = \sum\nolimits_{t=1}^{T} p_{ij}^t a_{ij}^t, \ \forall (i,j) \in E,$$
$$0 \le a_{ij}^t \le \Delta a_{ij}^t, \ \forall t = 1, 2, ..., T,$$

where a_{ij}^t is the new power variable. Finally, the coefficients p_t correspond to the slope of the straight line in each interval,

$$p_{ij}^t = \frac{(\bar{a}_{ij}^t)^2 - (\bar{a}_{ij}^{t-1})^2}{\Delta a_{ij}^t}, \ \forall t = 1, 2, ..., T,$$

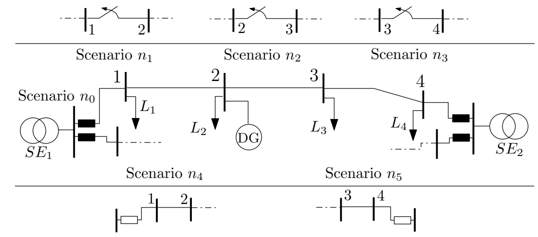


Fig. 3. Operating scenarios.

where it is always true that $p_{ij}^{t-1} a^{t-1} \le p_{ij}^t a^t$ since a_{ij}^2 is a non-decreasing function.

The resulting linear optimization problem is thus

$$\begin{aligned} \min \ & \sum\nolimits_{(i,j)\in E} \sum\nolimits_{t=1}^{T} \frac{R_{ij} l_{ij}}{(V_i \cos\theta_{ij})^2} p_{ij}^t a_{ij}^t \\ \text{s.t.} \ & \sum\nolimits_{j:(i,j)\in E} a_{ij}^t - \sum\nolimits_{j:(i,j)\in E} a_{ji}^t = b_i, \\ & \sum\nolimits_{t=1}^{T} a_{ij}^t \le u_{ij}, \ \forall (i,j) \in E \\ & 0 \le a_{ij}^t \le \Delta_{ij}^t. \end{aligned}$$

After solving the minimum power-loss problem, for each conductor type, we find a set of candidate primary feeders between any two substations. This process is repeated for every pair of substations.

B. Operating Scenarios

The next step is to define the operating scenarios, considering all the possible open and closed-loop arrangements along the main feeder. Figure 3 illustrates this step with the primary feeder linking two substations, SE_1 and SE_2. Scenario n_0 corresponds to a closed-loop arrangement, where the demand energy is supplied by two sources, and the switching elements (if any) have a normally closed state. Other scenarios are established by reconfiguring the network, changing the state of normally open and closed switches. Scenarios n_1 to n_3 in Figure 3 show these reconfigurations, in which the system operates with two primary feeders in an open-loop arrangement. Finally, scenarios n_4 and n_5 illustrate the case of failure in any of the substations, where the operation of the circuit breaker within the substation leads to the radial operation of a single primary feeder. These scenarios illustrate the load transfer between primary feeders. In this manner we find all the possible n_s scenarios for each candidate primary feeder, where $n_s = n_l + 1$, and n_l is the number of lines in the primary feeder.

C. Efficiency-based evaluation

For each candidate primary feeders and operation scenario, we obtain the power losses and the feeder's chargeability. The next step is to rank the candidate primary feeders following the approach in [7], which we now summarize and show how it is integrated into the proposed methodology. The efficiency-based evaluation relies on DEA, a deterministic non-parametric mathematical programming technique, which determines the technical efficiency of a set of decision-making units (DMUs). The efficiency of a DMU is evaluated by its ability to transform inputs into outputs [6]. In our methodology, the DMUs are the set F, with $n_F = |F|$, of all possible primary feeders candidates.

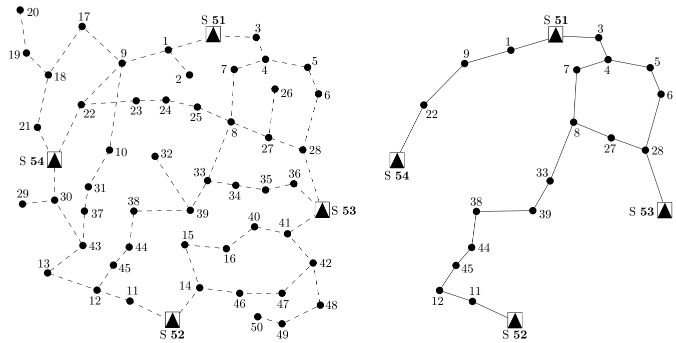
We use the DEA CCR-I (input-oriented) model [6], according to which a DMU is efficient if it maximizes the ratio of the weighted sum of its outputs divided by the weighted sum of its inputs. We consider the following three input variables: 1) capital costs, x_1; 2) power losses, x_2; and 3) the feeders' chargeability x_3. The output variable is a dummy equal to 1 for every DMU. To determine x_2 and x_3, and to consider the demand variability and the DG penetration into the analysis, we generate 100 random scenarios for each operation scenario and DMU. In each of these random scenarios we generate different demand values for each load node according to a uniform distribution, in an interval between the minimum and maximum peak demands expected during the planning horizon. We consider scenarios where the minimum demand can be negative, thus considering the effects that the DG power injection could cause, such as congestion or power discharge in certain network sections. We can evaluate different DG penetration levels using these scenarios, to evaluate the candidate feeders performance. However, it is important to consider that the probability of having voltage violations in load nodes increases as the DG penetration levels increase beyond the DS peak load. After this analysis, we obtain, for each DMU, x_2 as the 99-th percentile of the power losses, and x_3 as the average line chargeability.

Using the CCR-I DEA model, we obtain an efficiency measure θ_f for each DMU f, which is in the range $[0, 1]$, where 1 stands for full efficiency. As many DMUs can be ranked as fully-efficient, we use the super-efficiency model proposed in [11] to discriminate among the efficient DMUs. The best-ranked DMUs are the best primary feeders among the set of candidates.

III. Test and Results

We now illustrate the methodology on a test system [12] of 54 nodes (4 substations and 50 load nodes) and 64 branches operating under 13.2kV, depicted in Figure 4(a). We consider 5 conductor types, with different amperage capacities, for the installation of lines. The conductor type 5 has the largest capacity while conductor type 1 has the lowest. Conductors with a larger capacity are less susceptible to interference and have less internal resistance, thus enduring high currents at great distances, but being more expensive. Voltage regulation on load nodes is defined as $\pm 5\%$, and the voltage magnitude for substations is set at 1.00 pu, as technical constraints [1]. We assume a limit of 30% for the DG penetration rate to evaluate its effect on the network topology and to model the demand variability.

For illustration, Table I shows the results of applying the minimum power-loss problem, from substation S51 to other substations. The second column details the 15 candidate feeders obtained. The topology obtained with these candidate feeders is shown in Figure 4(b). Candidate feeders between substations S51-S52 and S51-S54 differ in the type of conductor, but not on the route. Instead, two different optimal routes linking S51 and S53 substations are found, depending on the type of conductor used.



(a) 54-Node test system (b) Candidate feeders from S51

Fig. 4. Candidate feeders by minimum cost flow method.

TABLE I
EFFICIENCY EVALUATION OF SOLUTIONS

DMU	Candidate feeder	Cond	CCR-I	SUPER-I
1	**51**-1-9-22-**54**	5	1	1.59368
2	**51**-3-4-7-8-33-39-38-44-45-12-11-**52**	3	1	1.31369
3	**51**-3-4-5-6-28-**53**	2	1	1.15530
4	**51**-1-9-22-**54**	1	1	1.08637
5	**51**-1-9-22-**54**	4	1	1.03604
6	**51**-1-9-22-**54**	2	1	1.02342
7	**51**-1-9-22-**54**	3	1	1.00447
8	**51**-3-4-7-8-33-39-38-44-45-12-11-**52**	4	0.99803	-
9	**51**-3-4-7-8-27-28-**53**	5	0.99791	-
10	**51**-3-4-5-6-28-**53**	1	0.93556	-
11	**51**-3-4-7-8-27-28-**53**	3	0.88277	-
12	**51**-3-4-7-8-27-28-**53**	4	0.84935	-
13	**51**-3-4-7-8-33-39-38-44-45-12-11-**52**	5	0.84568	-
14	**51**-3-4-7-8-33-39-38-44-45-12-11-**52**	2	0.34225	-
15	**51**-3-4-7-8-33-39-38-44-45-12-11-**52**	1	0.30291	-

The next step is to perform the efficiency-based evaluation of these candidate feeders. The results of the basic DEA model, in Table I, column CCR-I, show that 7 candidate feeders form the efficient production frontier (efficiency $\theta = 1$). For the primary feeder linking substations S51 and S53, the DMU 3, which corresponds to conductor type 2, shows the highest efficiency. For the primary feeder between substations S51 and S52, conductor type 3, associated to DMU 2, is the preferred choice. For the feeder linking substations S51 and S54, several options appear as efficient under the CCR-I model. We thus rely on the super-efficiency DEA model, column SUPER-I in Table I, to find that the DMU 1, associated to conductor type 5, is the best evaluated. The final network topology is shown in Figure 5.

Notice that, as shown in Figure 5, there is a common node (node 4) in the feeders between the substations S51-S52 and S51-S53, which connects two primary feeders. Therefore, we need to determine the open-loop arrangement under normal operational conditions to ensure a radial setup. To this end, we consider the power losses and the chargeability of the lines observed in the load flow analysis for the n_s scenarios evaluated. We then perform a relative frequency analysis of the scenarios with fewest power losses.

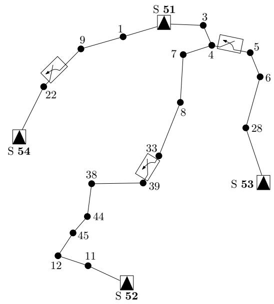


Fig. 5. Final feeders from S51.

TABLE II
RELATIVE FREQUENCY OF POWER LOSSES

Scenario	Open-loop	AbsFreq	RelFreq	Losses [MW]
1	3 4	10	0.1	0.0118
2	4 5	41	0.41	0.0086
3	5 6	28	0.28	0.0073
4	6 28	10	0.1	0.0134
5	S 51 1	10	0.1	0.0223
6	28 S 53	1	0.01	0.0172

For each operational scenario, we define its absolute frequency (AbsFreq) as the number of times that, across all demand conditions considered, it appears as the scenario with the fewest power losses. The relative frequency (RelFreq) is the absolute frequency divided by the number of demand condition considered. Table II shows the open-loop scenarios that can be formed with the feeder linking substations S52-S53, and, considering the 100 different demand conditions, the absolute and relative frequency for each scenario as well as the average power losses. Here, scenario 3 appears in 28 experiments with the fewest losses, with average power losses of 7.3×10^{-3} MW, while scenario 2 appears in 41 experiments with average power losses of 8.6×10^{-3} MW. In spite of the advantage for scenario 3 in average power losses, scenario 2 is more suitable for open-loop operation in high-demand scenarios, as it has the highest relative frequency. A similar analysis for feeders between substation S51-S54 and S51-S52, indicates the most suitable scenarios for the final topology, depicted in Figure 5.

We have conducted a similar analysis for the closed-loop scenarios to assess their performance against the open-loop arrangements for different levels of DG penetration. As shown in Table III, with a DG penetration rate of 30 %, as in the above analysis, scenario 0 has the highest relative frequency. Also, as the DG penetration increases, the relative frequency of scenario 0 also increases, while the relative frequency of the open-loop scenarios decreases, making the latter even more unsuitable for system operation. From these results it is clear that the total load-transfer arrangements, scenarios 5 and 6, have the higher average power losses, which is consistent with the fact that these arrangements operate under extreme contingency conditions.

IV. CONCLUSION

This paper introduces a methodology for the planning of primary feeders, which takes into account open and closed-loop operational scenarios as well as DG penetration. This is done by initially finding a set of candidate primary feeders by solving a minimum power-loss problem, considering technical and topological features. Then, we use different operational scenarios and efficiency-based evaluation to define the most suitable arrangements for open and closed-loop operation. The proposed methodology is therefore able to find primary feeders that are well-suited for open and closed-loop operation, ensuring good performance of the DS under normal and contingency operational conditions. The proposed mesh design allows the DS to operate in open-loop arrangements, fitting well within smart-grid schemes, where the operating conditions continually change. The open-loop arrangement ensures a radial operation, whereas an increasing DG penetration makes the closed-loop arrangement a more convenient option.

TABLE III
RELATIVE FREQUENCY OF POWER LOSSES VS DG PENETRATION

DG penetration		10%	15%	20%	25%	30%
Scenario	**Topology**	**Relative frequency of power losses**				
0	S 51 1 28 S 53	0.29	0.34	0.38	0.52	0.61
1	3 4	0.10	0.08	0.09	0.05	0.07
2	4 5	0.28	0.25	0.23	0.15	0.16
3	5 6	0.22	0.18	0.18	0.17	0.06
4	6 28	0.07	0.06	0.08	0.06	0.04
5	S 51 1	0.02	0.03	0.01	0.02	0.01
6	28 S 53	0.02	0.06	0.03	0.03	0.05

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